第四届“偏微分方程数值方法与理论暑期学校”

2024 年 8 月 10-29 日

中国科学技术大学数学科学学院
安徽应用数学中心
数字技术的数学基础与应用教育部重点实验室

第四届“偏微分方程数值方法与理论暑期学校”

“偏微分方程数值方法与理论暑期学校”由国内二十多所高校和研究所共同发起，其宗旨是通过开设暑期课程、专题讲座和举办前沿学术研讨会，探索青年人才培养的新模式，为国内建设和培养研究视野宽、创新意识强、数学基础深厚、计算能力坚实、有国际影响力的计算数学与应用数学人才队伍做贡献，从而推动和促进国内及国际上偏微分方程数值方法与理论的发展。前三届“偏微分方程数值方法与理论暑期学校”分别在华南师范大学（2017 年 7 月 31 日至 8 月 16 日）、北京工业大学-北京科学与工程计算研究院（2018 年 7 月 30 日至 8 月 18 日）和四川大学（2019 年 7 月 23 日至 8 月 11 日）成功举办。

“第四届偏微分方程数值方法与理论暑期学校”由中国科学技术大学承办。本次暑期学校将于 2024 年 8 月 10 日至 8 月 27 日开设三门暑期课程和多个专题讲座，于 8 月 28 日举办学术前沿讨论会。

主办单位

中国科学技术大学数学科学学院、安徽应用数学中心、数字技术的数学基础与应用教育部重点实验室

学术委员会（按姓氏字母排序）

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许进超	King Abdullah University of Science and Technology
邹　军	香港中文大学

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招生对象 青年教师及研究生

联系人

卢珊珊 电话：0551-63601434 电子邮箱：compustc@163.com

暑期课程安排

（东区第五教学楼 5103 教室，每日 9:30-11:30）

1、8 月 11 日-8 月 15 日，Finite Element Methods for Eigenvalue Problems，Daniele Boffi（King Abdullah University of Science and Technology）
2、8 月 17 日-8 月 21 日，Discontinuous Galerkin Finite Element Methods，J.J.W. van der Vegt (University of Twente)
3、8 月 22 日-8 月 23 日，8 月 25 日-8 月 27 日， Introduction to Multigrid Methods, J.J.W. van der Vegt (University of Twente)

前沿讲座安排

（东区第五教学楼 5103 教室，每日 15:00-17:00）

1. 8 月 11 日 胡俊（北京大学）
2. 8 月 12 日 戴小英（中国科学院数学与系统科学研究院）
3. 8 月 13 日 赵泉（中国科学技术大学）
4. 8 月 14 日 张智民（韦恩州立大学）
5. 8 月 15 日 徐宽（中国科学技术大学）
6. 8 月 17 日 杜洁（华东师范大学）
7. 8 月 18 日 谭志裕（厦门大学）
8. 8 月 19 日 陈黄鑫（厦门大学）
9. 8 月 20 日 吴朔男（北京大学）
10. 8 月 21 日 张智民（韦恩州立大学）
11. 8 月 22 日 徐岩（中国科学技术大学）
12. 8 月 23 日 徐岩（中国科学技术大学）
13. 8 月 25 日 乔中华（香港理工大学）
14. 8 月 26 日 吴开亮（南方科技大学）
15. 8 月 27 日 陈景润（中国科学技术大学）

8 月 28 日日程安排

（东区第五教学楼 5103 教室）

9:00-12:10 学术前沿讨论会

报告人：唐庆粦（四川大学）
王翔（吉林大学）
徐岩（中国科学技术大学）

15:00-17:00 考试

偏微分方程数值方法与理论暑期学校日程安排

（2024 年 8 月 10 日-8 月 29 日，东区第五教学楼 5103）

月	星期日		星期一		星期二		星期三		星期四		星期五		星期六	
八		9:10 暑期学校开幕式											10	暑期学校报到
	11	9:30-11:30 Daniele Boffi 特征值问题有限元方法	12	9:30-11:30 Daniele Boffi 特征值问题有限元方法	13	9:30-11:30 Daniele Boffi 特征值问题有限元方法	14	9:30-11:30 Daniele Boffi 特征值问题有限元方法	15	9:30-11:30 Daniele Boffi 特征值问题有限元方法	16	休息	17	9:30-11:30 van der Vegt 间断有限元
		15:00-17:00 前沿讲座 胡俊		15:00-17:00 前沿讲座 戴小英		15:00-17:00 前沿讲座 赵泉		15:00-17:00 前沿讲座 张智民		15:00-17:00 前沿讲座 徐宽				15:00-17:00 前沿讲座 杜洁
	18	9:30-11:30 van der Vegt 间断有限元	19	9:30-11:30 van der Vegt 间断有限元	20	9:30-11:30 van der Vegt 间断有限元	21	9:30-11:30 van der Vegt 间断有限元	22	9:30-11:30 van der Vegt 多重网格方法	23	9:30-11:30 van der Vegt 多重网格方法	24	休息
		15:00-17:00 前沿讲座 谭志裕		15:00-17:00 前沿讲座 陈黄鑫		15:00-17:00 前沿讲座 吴朔男		15:00-17:00 前沿讲座 张智民		15:00-17:00 前沿讲座 徐岩		15:00-17:00 前沿讲座 徐岩		
	25	9:30-11:30 van der Vegt 多重网格方法	26	9:30-11:30 van der Vegt 多重网格方法	27	9:30-11:30 van der Vegt 多重网格方法	28	前沿研讨会	29	暑期学校离会				
		15:00-17:00 前沿讲座 乔中华		15:00-17:00 前沿讲座 吴开亮		15:00-17:00 前沿讲座 陈景润		考试						

暑期课程摘要

Finite Element Methods for Eigenvalue Problems

Daniele Boffi (King Abdullah University of Science and Technology)

Eigenvalue problems are important in many applications areas, such as electromagnetics, structural mechanics, quantum theory, and control. Important examples are the Laplace, Maxwell and Schrodinger eigenvalue problems. Finite element methods provide a well-established numerical method to solve eigenvalue problems and are supported by an extensive mathematical theory. In this short course, we will discuss several finite element methods for the computation of eigenvalue problems. Special attention will be given to the theoretical analysis of finite element discretizations for eigenvalue problems. In particular, the convergence theories from Descloux, Nassif and Rapaz, and from Babuska and Osborn will be discussed and used to analyze the convergence of finite element discretizations for eigenvalue problems.
Reference: D. Boffi, Finite element approximation of eigenvalue problems, Acta Numerica, 19 (2010), pp. 1–120.

Discontinuous Galerkin Finite Element Methods

J.J.W. van der Vegt (University of Twente)

Discontinuous Galerkin (DG) finite element methods are nowadays one of the main numerical techniques to solve partial differential equations. The key feature of DG methods is that discontinuities are allowed in the test and trial spaces at element faces. This provides great flexibility to build higher order accurate, solution adaptive numerical discretizations, using local mesh refinement and the local adjustment of the polynomial order of the test and trial spaces. DG methods also allow for efficient parallel computing due to minimal element connectivity, and provide element wise conservative numerical discretizations, which is especially important for hyperbolic partial differential equations. In these lectures we will discuss the basic principles of discontinuous Galerkin methods for several important classes of partial differential equations (hyperbolic, elliptic). Special attention will be given to the mathematical aspects of DG methods, such as stability, convergence, and accuracy by studying and analyzing several model problems in detail. If time permits also, the extension to space-time DG discretizations, which use discontinuous test and trial functions in space and in time, will be considered for the advection equation and the incompressible Navier-Stokes equations. As pre-existing knowledge for this course familiarity with standard conforming finite element methods and their analysis is assumed.
References: D.A. Di Pietro, A. Ern, Mathematical aspects of discontinuous Galerkin methods, Springer 2012, ISBN 978-3-642-22979-4.

Introduction to Multigrid Methods

Professor J.J.W. van der Vegt (University of Twente)

Multigrid methods provide very efficient iterative methods for the solution of large systems of (non)linear algebraic equations that result for instance from the discretization of partial differential equations. In a multigrid method several coarsened approximations of the algebraic system and well-designed smoothers are used to accelerate the convergence of the iterative method. This can result in very efficient iterative methods, but if one wants to develop new multigrid algorithms or understand the performance of existing algorithms, then multilevel analysis is indispensible. In this class an outline of basic multigrid and iterative methods will be given and mathematical techniques to understand and predict their performance will be discussed. No prior knowledge of multigrid or iterative methods will be required. After this class you should be able to use basic iterative and multigrid methods, analyze and (approximately) predict multigrid performance using multilevel analysis and apply these techniques to improve and test multigrid algorithms. The main applications will be from numerical discretizations of partial differential equations.

Reference: U. Trottenberg, C.W. Oosterlee, A. Schüller, Multigrid, Academic Press, ISBN 0-12-701070-X, 2001.

前沿讲座摘要

1. **8 月 11 日 胡俊（北京大学）**
   报告题目：弹性力学问题混合有限元方法
   摘要: 有限元方法是在求解弹性力学问题过程中发明的，其最重要的应用是弹性应力分析。混合有限元方法可以直接计算弹性应力，比较精确地满足平衡方程和应力边界条件，克服位移有限元方法的闭锁现象，是弹性力学问题非常重要的数值方法。但是其构造却是一个有 50 多年历史的公开问题。其中最大困难在于离散应力与位移空间对既要满足混合有限元方法的稳定性,又要满足离散应力的对称性,还要满足空间对匹配的完美性。很多学者都为此努力过，包括国际数学家大会一小时大会报告人 D. N. Arnold 和 F. Brezzi。本报告将介绍最近构造的应力空间和位移空间完美匹配的弹性力学问题稳定的混合有限元方法。

2. **8 月 12 日 戴小英（中国科学院数学与系统科学研究院）**
   报告题目：特征值问题的自适应离散
   摘要：科学工程计算中很多问题都归结为微分算子特征值问题的计算。如何对微分算子特征值问题进行高效离散是一具有重要科学意义的研究课题。本报告将从电子结构模型出发，介绍微分算子特征值问题在其中的核心作用及其自适应离散，包括自适应有限元离散和自适应平面波离散。

3. **8 月 13 日 赵泉（中国科学技术大学）**
   报告题目：Structure-preserving parametric finite element methods for geometric PDEs with applications
   摘要：The motion of interfaces driven by a law for the normal velocity which involves curvature quantities plays an important role in materials sciences, interface dynamics in multiphase flow and applied mathematics, etc. In this talk, I will present a structure-preserving parametric finite element method (PFEM) for the geometric flow and then generalize the introduced method to the multiphase flow. Finally, applications to dewetting of solid thin films with sharp-interface models and PFEM approximations will be discussed.

4. **8 月 14 日 张智民（韦恩州立大学）**
   报告题目：Why spectral methods are preferred in PDE eigenvalue computations --- in some cases?
   摘要：When approximating PDE eigenvalue problems by numerical methods such as finite difference and finite element methods, it is well-known that only a small portion of the numerical eigenvalues are reliable. In contrast, spectral methods can perform extremely well in some situations, especially for one-dimensional problems and certain special higher-dimensional cases. Furthermore, we demonstrate that spectral methods can outperform traditional methods and the state-of-the-art methods for two-dimensional problems, even those with singularities.

5. 8 月 15 日 徐宽（中国科学技术大学）

报告题目：The ultraspherical spectral method

摘要：超球谱方法（ultraspherical spectral method）是一种新诞生的求解各类微分方程、积分方程的谱方法。它具有速度快、精度高、稳定性强、易自适应的特点。超球谱方法将在未来取代几乎现有的所有谱方法。本报告将介绍这一方法和它最近的发展。

6. 8 月 17 日 杜洁（华东师范大学）

报告题目：High order bound preserving and oscillation free methods for compressible multi-species flow with chemical reactions

摘要：In this talk, we consider bound preserving problems for multispecies and multireaction chemical reactive flows. In this problem, the density and pressure are nonnegative, and the mass fraction should be between 0 and 1. The mass fraction does not satisfy a maximum principle and hence it is not easy to preserve the upper bound 1. Also, most of the bound-preserving techniques available are based on Euler forward time integration. Therefore, for problems with stiff source, the time step will be significantly limited. Some previous ODE solvers for stiff problems cannot preserve the total mass and the positivity of the numerical approximations at the same time. In this work, we will construct third order conservative bound-preserving methods to overcome all these difficulties. Moreover, we will discuss how to control numerical oscillations.

7. 8 月 18 日 谭志裕（厦门大学）

报告题目：Nonlinear least-squares finite element methods for fully nonlinear partial differential equations

摘要：In this talk, I will introduce nonlinear least-squares finite element methods for fully nonlinear PDEs based on the C^0 interior penalty method and mainly focus on the methods for the Monge-Ampère equation and the Pucci equation on convex domains in 2D.

8. 8 月 19 日 陈黄鑫（厦门大学）

报告题目：Efficient and physics-preserving numerical methods for two-phase flow and multicomponent flow in porous media

摘要：Modeling and simulation of two-phase flow and gas flow in porous media are of great interest in the fields of hydrology and petroleum reservoir engineering. In this talk, we will first introduce a new physics-preserving IMPES scheme for simulating classical incompressible and immiscible two-phase flow in heterogeneous porous media with capillary pressure effects. The new algorithm is locally mass conservative for both phases. Furthermore, the scheme is unbiased with respect to the two phases, and the saturations of both phases are bounds-preserving provided the time step size is smaller than a certain threshold value. We will then apply this new approach to develop a fully mass-conservative IMPEC scheme for modeling classical multicomponent flow in porous media. Additionally, we will discuss the recently developed thermodynamically consistent modeling and simulation for two-phase flow and multicomponent flow in porous media.

9. 8月20日 吴朔男（北京大学）

报告题目：对流-扩散方程的稳定化有限元方法

摘要：对流-扩散方程作为多物理场耦合的基本模型之一，广泛应用于各个领域。然而，当对流占优时，边界层的出现会导致传统有限元方法在边界层内失去稳定性，从而产生剧烈震荡。本讲座将从标量对流-扩散方程入手，介绍两类在拟均匀网格下的稳定化有限元离散方法：迎风型方法和指数拟合方法。迎风型方法通过在变分形式中引入对流速度信息来增加稳定性，而指数拟合方法则利用边界层解的特征，将指数函数融入到格式设计中。近年来，对流-扩散方程在电磁场等向量场问题中的应用日益重要，其对流项的数学形式和结构也变得更加复杂。本讲座将探讨上述两类稳定化方法在这些新型向量场问题中的推广应用。

10. 8月21日 张智民（韦恩州立大学）

报告题目：局部间断有限元的超收敛

摘要：The phenomenon of superconvergence is well understood for the h-version finite element method, and researchers in this established field have accumulated a vast body of literature over the past 60 years. However, there is a lack of relevant systematics studies for other numerical methods such as the p-version finite element method, spectral methods, discontinuous Galerkin methods, and finite volume methods. We believe that the scientific community would also benefit from studying of superconvergence phenomenon in these methods. In the last decade, efforts have been made to expand the scope of superconvergence. In this talk, we present some developments in the study of superconvergence for the local discontinuous Galerkin methods.

11. 8月22日 徐岩（中国科学技术大学）

报告题目：Discontinuous Galerkin method for the nonlinear time-dependent equations

摘要：In this talk, we discuss local discontinuous Galerkin method for solving the nonlinear time-dependent equations which contain nonlinear high order derivatives. The discretization results in an extremely local, element based discretization, which is beneficial for parallel computing and maintaining high order accuracy on unstructured meshes. In particular, the methods are well suited for hp-adaptation, which consists of local mesh refinement and/or the adjustment of the polynomial order in individual elements. The stability and the error estimates of the numerical methods will be discussed. Numerical simulation results for different types of solutions illustrate the accuracy and capability of the methods.

12. 8月23日 徐岩（中国科学技术大学）

报告题目：Higher order accurate bounds preserving time-implicit discretizations for the nonlinear time-dependent equations

摘要：In this talk, we discuss a novel semi-implicit spectral deferred correction (SDC) time marching method. The method can be used in a large class of problems, especially for highly nonlinear ordinary differential equations (ODEs) without easily separating of stiff and non-stiff components, which is more general and efficient comparing with traditional semi-implicit SDC methods. The proposed semi-implicit SDC method is based on low order time integration methods and corrected iteratively. The order of accuracy is increased for each additional iteration. This SDC method is intended to be combined with the method of lines, which provides a flexible framework to develop high order semi-implicit time marching methods for nonlinear partial differential equations (PDEs). Coupled with the LDG spatial discretization, the fully discrete schemes are all high order accurate in both space and time, and stable numerically with the time step proportional to the spatial mesh size. Using Lagrange multipliers the conditions imposed by the positivity preserving limiters are directly coupled to a DG discretization combined with implicit time integration method. The positivity preserving DG discretization is then reformulated as a Karush-Kuhn-Tucker (KKT) problem. We therefore develop an efficient active set semi-smooth Newton method that is suitable for the KKT formulation of time-implicit positivity preserving DG discretizations. Convergence of this semi-smooth Newton method is proven using a specially designed quasi-directional derivative of the time-implicit positivity preserving DG discretization. Numerical experiments are carried out to illustrate the accuracy and capability of the proposed method.

13. 8 月 25 日 乔中华（香港理工大学）

报告题目：相场方程及其高效数值方法

摘要：相场方法是求解界面问题的一种有效的数学方法。在实际应用中，可以通过构造不同的自由能泛函来建立相应的相场模型。计算机模拟在对相场模型的研究中起着非常重要的作用。虽然发展稳定、高效的数值格式来求解相场模型已得到了人们持续关注，但是相场模型的数值模拟仍旧是具有挑战性的课题，特别是在高维问题模拟中，因为涉及的计算量非常巨大，困难变得尤其突出。我们将回顾一些常见的相场方程及相关的数值方法，并且讨论最新的一些研究进展。

14. 8 月 26 日 吴开亮（南方科技大学）

报告题目：Bound-Preserving Schemes and Theory

摘要：Solutions to many partial differential equations (PDEs) are subject to certain bounds or constraints. For instance, in fluid dynamics, density and pressure must remain positive, while in relativistic cases, fluid velocity must not exceed the speed of light. Developing bound-preserving numerical methods that uphold these intrinsic constraints is crucial. Recently, significant attention has been given to design provably bound-preserving schemes, though challenges remain, particularly for systems with nonlinear constraints.In this talk, I will present our recent efforts in developing foundemental bound-preserving theories:

1. Geometric Quasilinearization (GQL): Drawing on key insights from geometry, we propose a novel and general framework called geometric quasilinearization. GQL offers an effective approach for addressing bound-preserving problems with nonlinear constraints by transforming these constraints into linear ones through the introduction of auxiliary variables. We establish the fundamental principles and general theory of GQL using the geometric properties of convex regions and present three effective methods for constructing GQL.

2. Optimal Cell Average Decomposition (OCAD): Utilizing convex geometry and symmetric group theory, we develop the optimal cell average decomposition theory, which provides a foundation for constructing more efficient bound-

preserving schemes. We demonstrate that the classic Zhang-Shu CAD is optimal in one dimension but generally not in multiple dimensions, thereby addressing their conjecture proposed in 2010.
We apply the GQL and OCAD approaches to various PDEs, showcasing their effectiveness and advantages through diverse and challenging examples and applications, including magnetohydrodynamics (MHD), relativistic hydrodynamics, and the ten-moment Gaussian closure system.

15. **8 月 27 日 陈景润（中国科学技术大学）**
报告题目：科学计算：传统方法 vs 机器学习方法
摘要：如何数值求解偏微分方程是科学与工程领域的重要问题。传统方法包括有限差分、有限元等。近年来深度学习也被利用来求解偏微分方程。本报告将从精度和效率两个方面，系统比较传统方法和机器学习方法的优缺点。时间允许的话，本报告还会讨论工业设计智能化对科学计算算法的要求。

学术前沿讨论会报告摘要

唐庆粦（四川大学）

报告题目：A preconditioned Riemannian conjugate gradient method to compute the ground state of arbitrary-angle rotating dipolar Bose-Einstein Condensates

摘要：In this talk, we will present an efficient numerical method for computing the ground state of the arbitrary-angle rotating dipolar Bose-Einstein Condensates (BEC). The method consists two main merits: (i) efficient and accurate numerical methods will be proposed to evaluate the nonlocal dipole-dipole interaction. (ii) a preconditioned Riemannian conjugate gradient (pRCG) method coupled with the Fourier pseudo-spectral discretization to numerically compute the GS. A robust and efficient preconditioner together with an adaptive stepsize control strategy are provided to accelerate the convergence process of pRCG algorithm. Utilizing the pRCG algorithm, GS patterns of arbitrary- angle rotating BEC under various settings are carried out. Particularly, for the first time, we observe bent vortex lines, including U-shape and S-shape vortex lines, in the elongated BEC with arbitrary-angle rotation.

王翔（吉林大学）

报告题目：The finite volume element method with global conservation law

摘要：Conservation laws are fundamental physical properties that are expected to be preserved in numerical discretizations. We propose a two-layered dual strategy for the finite volume element method (FVEM), which possesses the conservation laws in both flux form and equation form. In particular, for problems with Dirichlet boundary conditions, the proposed schemes preserves conservation laws on all triangles, whereas conservation properties may be lost on boundary dual elements by existing vertex-centered finite volume element schemes. Theoretically, we carry out the optimal L^2 analysis with reducing the regularity requirement from $u\in H^{k+2}$ to $u\in H^{k+1}$. While, as a comparison, all existing L^2 results for high-order $(k>=2)$ finite volume element schemes require $u\in H^{k+2}$ in the analysis.

徐岩（中国科学技术大学）

报告题目：Structure-preserving arbitrary Lagrangian-Eulerian discontinuous Galerkin methods for hyperbolic conservation law with source term

摘要：We develop the structure-preserving Lagrangian-Eulerian discontinuous Galerkin (ALE-DG) methods for a class of hyperbolic conservation laws with source term, which can preserve a general hydrostatic equilibrium state and positivity-preserving property under a suitable time step at the same time. Such equations mainly include the shallow water equations with non-flat bottom topography and the Euler equations with gravitation. By introducing well-balanced numerical fluxes and corresponding source term approximations, we established well-balanced schemes. We also discuss about the weak positivity property of the proposed schemes, and the positivity-preserving limiter can be applied to enforce the positivity-preserving property effectively. Numerical examples have been provided not only to demonstrate the good properties but also to show the advantages on moving mesh.